Symmetry Limit Properties of A Priori Mixing Amplitudes for Non-leptonic and Weak Radiative Decays of Hyperons

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Abstract

We show that the so-called parity-conserving amplitudes predicted in the a priori mixing scheme for non-leptonic and weak radiative decays of hyperons vanish in the strong-flavor symmetry limit.

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The presence of a priori mixings of strong-flavor and parity eigenstates in physical (mass eigenstate) hadrons provides the basis for an explanation of the enhancement phenomenon observed in non-leptonic and weak radiative decays of hyperons (NLDH and WRDH), as we have shown elsewhere [1–3]. The transition operators in the amplitudes of these decays are the parity and flavor conserving strong-interaction Yukawa hamiltonian H_Y and the ordinary electromagnetic hamiltonian H_{em}. Although both of them break the strong-flavor symmetries, it is nevertheless interesting to consider the consequences of assuming that they do not, i. e., the consequences on NLDH and WRDH of the symmetry limit behavior of these two operators. In this paper we shall demostrate a theorem that says that the so-called parity-conserving amplitudes in NLDH and WRDH [4] obtained in the a priori mixing scheme vanish in the strong-flavor symmetry limit.

Let us start with NLDH. The expressions for the physical hadrons that concern us here in terms of a priori mixings are [1]

$$
\begin{aligned}
K^+_{ph} &= K^+_{0p} - \sigma\pi^+_{0p} - \delta'\pi^+_{0s} + \cdots, \\
K^0_{ph} &= K^0_{0p} + \frac{1}{\sqrt{2}}\sigma\pi^0_{0p} + \frac{1}{\sqrt{2}}\delta'\pi^0_{0s} + \cdots, \\
\pi^+_{ph} &= \pi^+_{0p} + \sigma K^+_{0p} - \delta K^+_{0s} + \cdots, \\
\pi^0_{ph} &= \pi^0_{0p} - \frac{1}{\sqrt{2}}\sigma(K^0_{0p} + \bar{K}^0_{0p}) + \frac{1}{\sqrt{2}}\delta(K^0_{0s} - \bar{K}^0_{0s}) + \cdots, \\
\pi^-_{ph} &= \pi^-_{0p} + \sigma K^-_{0p} + \delta K^-_{0s} + \cdots, \\
\bar{K}^0_{ph} &= \bar{K}^0_{0p} + \frac{1}{\sqrt{2}}\sigma\pi^0_{0p} - \frac{1}{\sqrt{2}}\delta'\pi^0_{0s} + \cdots, \\
K^-_{ph} &= K^-_{0p} - \sigma\pi^-_{0p} + \delta'\pi^-_{0s} + \cdots,
\end{aligned}
\tag{1}
$$

and,

$$
\begin{aligned}
p_{ph} &= p_{0s} - \sigma\Sigma^+_{0s} - \delta\Sigma^+_{0p} + \cdots, \\
n_{ph} &= n_{0s} + \sigma(\frac{1}{\sqrt{2}}\Sigma^0_{0s} + \sqrt{\frac{3}{2}}\Lambda_{0s}) + \delta(\frac{1}{\sqrt{2}}\Sigma^0_{0p} + \sqrt{\frac{3}{2}}\Lambda_{0p}) + \cdots, \\
\Sigma^+_{ph} &= \Sigma^+_{0s} + \sigma p_{0s} - \delta' p_{0p} + \cdots, \\
\Sigma^0_{ph} &= \Sigma^0_{0s} + \frac{1}{\sqrt{2}}\sigma(\Xi^0_{0s} - n_{0s}) + \frac{1}{\sqrt{2}}\delta\Xi^0_{0p} + \frac{1}{\sqrt{2}}\delta' n_{0p} + \cdots,
\end{aligned}
\tag{2}
$$

$$\Sigma^-_{ph} = \Sigma^-_{0s} + \sigma\Xi^-_{0s} + \delta\Xi^-_{0p} + \cdots,$$

$$\Lambda_{ph} = \Lambda_{0s} + \sqrt{\frac{3}{2}}\sigma(\Xi^0_{0s} - n_{0s}) + \sqrt{\frac{3}{2}}\delta\Xi^0_{0p} + \sqrt{\frac{3}{2}}\delta' n_{0p} + \cdots,$$

$$\Xi^0_{ph} = \Xi^0_{0s} - \sigma(\frac{1}{\sqrt{2}}\Sigma^0_{0s} + \sqrt{\frac{3}{2}}\Lambda_{0s}) + \delta'(\frac{1}{\sqrt{2}}\Sigma^0_{0p} + \sqrt{\frac{3}{2}}\Lambda_{0p}) + \cdots,$$

$$\Xi^-_{ph} = \Xi^-_{0s} - \sigma\Sigma^-_{0s} + \delta'\Sigma^-_{0p} + \cdots.$$

The dots stand for other mixings that will not be relevant here. The subindeces naught, s, and p mean flavor, positive, and negative parity eigenstates, respectively. The amplitudes for NLDH are of the form $\bar{u}_{B'}(A+B\gamma_5)u_B$, where A and B are the so-called parity-violating and parity-conserving amplitudes, respectively. In what follows, we shall need the expressions for the B's only, they are given by [1]

$$B_1 = \sigma(-\sqrt{3}g_{_{p,p\pi^0}} + g_{_{\Lambda,pK^-}} - g_{_{\Lambda,\Sigma^+\pi^-}}),$$

$$B_2 = -\frac{1}{\sqrt{2}}\sigma(-\sqrt{3}g_{_{p,p\pi^0}} + g_{_{\Lambda,pK^-}} - g_{_{\Lambda,\Sigma^+\pi^-}}),$$

$$B_3 = \sigma(\sqrt{2}g_{_{\Sigma^0,pK^-}} + \sqrt{\frac{3}{2}}g_{_{\Sigma^+,\Lambda\pi^+}} + \frac{1}{\sqrt{2}}g_{_{\Sigma^+,\Sigma^+\pi^0}}),$$

$$B_4 = \sigma(\sqrt{2}g_{_{p,p\pi^0}} + \sqrt{\frac{3}{2}}g_{_{\Sigma^+,\Lambda\pi^+}} - \frac{1}{\sqrt{2}}g_{_{\Sigma^+,\Sigma^+\pi^0}}), \tag{3}$$

$$B_5 = \sigma(g_{_{p,p\pi^0}} - g_{_{\Sigma^0,pK^-}} - g_{_{\Sigma^+,\Sigma^+\pi^0}}),$$

$$B_6 = \sigma(-g_{_{\Sigma^+,\Lambda\pi^+}} + g_{_{\Xi^-,\Lambda K^-}} + \sqrt{3}g_{_{\Xi^0,\Xi^0\pi^0}}),$$

$$B_7 = \frac{1}{\sqrt{2}}\sigma(-g_{_{\Sigma^+,\Lambda\pi^+}} + g_{_{\Xi^-,\Lambda K^-}} + \sqrt{3}g_{_{\Xi^0,\Xi^0\pi^0}}).$$

The subindeces $1,\ldots,7$ correspond to $\Lambda \to p\pi^-$, $\Lambda \to n\pi^0$, $\Sigma^- \to n\pi^-$, $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$, $\Xi^- \to \Lambda\pi^-$, and $\Xi^0 \to \Lambda\pi^0$, respectively. Here σ is the a priori mixing angle that accompanies the positive-parity eigenstates in the physical hadrons. Notice the numerical coefficients in these B's. They are the ones that accompany σ in the physical hadrons. The coupling constants that appear in Eqs. (3) are the ordinary Yukawa couplings observed in strong interactions. To demostrate the theorem for NLDH we must assume that H_Y is an

invariant operator, an SU_3 invariant in the case of Eqs. (3). In this limit the g's are given by [5]

$$g_{_{p,p\pi^0}} = g, \qquad g_{_{\Sigma^+,\Lambda\pi^+}} = -\frac{2}{\sqrt{3}}\alpha g,$$

$$g_{_{\Lambda,\Sigma^+\pi^-}} = -\frac{2}{\sqrt{3}}\alpha g, \qquad g_{_{\Sigma^+,\Sigma^+\pi^0}} = 2(1-\alpha)g, \tag{4}$$

$$g_{_{\Sigma^0,pK^-}} = (2\alpha-1)g, \qquad g_{_{\Lambda,pK^-}} = \frac{1}{\sqrt{3}}(3-2\alpha)g,$$

$$g_{_{\Xi^0,\Xi^0\pi^0}} = -(2\alpha-1)g, \qquad g_{_{\Xi^-,\Lambda K^-}} = \frac{1}{\sqrt{3}}(4\alpha-3)g.$$

The connection between α and g and the reduced from factors F and D are $\alpha = D/(D+F)$ and $g = D + F$.

When the symmetry limit values of the g's, Eqs. (4), are replaced in Eqs. (3), one readily sees that each one of the B's become zero, i. e., $B_1 = B_2 = \cdots = B_7 = 0$, and thus the theorem follows for NLDH.

For WRDH the transition amplitudes are of the form $\bar{u}_{B'}(C + D\gamma_5)i\sigma^{\mu\nu}q_\nu u_B \epsilon_\mu$, where C and D are the so-called parity-conserving and parity-violating amplitudes, respectively. ϵ_μ is the polarization four-vector of the photon. The hadronic parts of the C and D amplitudes are given by [1]

$$\langle p_{ph}|J^\mu_{em}|\Sigma^+_{ph}\rangle = \bar{u}_p[\sigma(f_2^{\Sigma^+} - f_2^p) + (\delta' f_2^p - \delta f_2^{\Sigma^+})\gamma^5]i\sigma^{\mu\nu}q_\nu u_{\Sigma^+},$$

$$\langle \Sigma^-_{ph}|J^\mu_{em}|\Xi^-_{ph}\rangle = \bar{u}_{\Sigma^-}[\sigma(f_2^{\Xi^-} - f_2^{\Sigma^-}) + (\delta' f_2^{\Sigma^-} - \delta f_2^{\Xi^-})\gamma^5]i\sigma^{\mu\nu}q_\nu u_{\Xi^-},$$

$$\begin{aligned}\langle n_{ph}|J^\mu_{em}|\Lambda_{ph}\rangle = \bar{u}_n \Bigg\{ & \sigma\left[\sqrt{\frac{3}{2}}(f_2^\Lambda - f_2^n) + \frac{1}{\sqrt{2}}f_2^{\Sigma^0\Lambda}\right] \\ & + \left[\sqrt{\frac{3}{2}}(\delta' f_2^n - \delta f_2^\Lambda) - \delta\frac{1}{\sqrt{2}}f_2^{\Sigma^0\Lambda}\right]\gamma^5\Bigg\} i\sigma^{\mu\nu}q_\nu u_\Lambda,\end{aligned} \tag{5}$$

$$\begin{aligned}\langle \Lambda_{ph}|J^\mu_{em}|\Xi^0_{ph}\rangle = \bar{u}_\Lambda \Bigg\{ & \sigma\left[\sqrt{\frac{3}{2}}(f_2^{\Xi^0} - f_2^\Lambda) - \frac{1}{\sqrt{2}}f_2^{\Sigma^0\Lambda}\right] \\ & + \left[\sqrt{\frac{3}{2}}(\delta' f_2^\Lambda - \delta f_2^{\Xi^0}) + \delta'\frac{1}{\sqrt{2}}f_2^{\Sigma^0\Lambda}\right]\gamma^5\Bigg\} i\sigma^{\mu\nu}q_\nu u_{\Xi^0},\end{aligned}$$

$$\langle \Sigma^0_{ph} | J^\mu_{em} | \Xi^0_{ph} \rangle = \bar{u}_{\Sigma^0} \left\{ \sigma \left[\frac{1}{\sqrt{2}} (f_2^{\Xi^0} - f_2^{\Sigma^0}) - \sqrt{\frac{3}{2}} f_2^{\Sigma^0 \Lambda} \right] \right.$$
$$\left. + \left[\frac{1}{\sqrt{2}} (\delta' f_2^{\Sigma^0} - \delta f_2^{\Xi^0}) + \delta' \sqrt{\frac{3}{2}} f_2^{\Sigma^0 \Lambda} \right] \gamma^5 \right\} i\sigma^{\mu\nu} q_\nu u_{\Xi^0}.$$

Here J^μ_{em} is the electromagnetic current operator. The origin of the numerical coefficients is the same as in Eqs. (3). In the SU_3 symmetry limit the anomalous magnetic moments f_2 are related by

$$f_2^{\Sigma^+} = f_2^p, \quad f_2^{\Xi^-} = f_2^{\Sigma^-}, \quad f_2^{\Xi^0} = f_2^n, \tag{6}$$

$$f_2^{\Sigma^0 \Lambda} = \frac{\sqrt{3}}{2} f_2^n, \quad f_2^{\Sigma^0} = -\frac{1}{2} f_2^n, \quad f_2^{\Lambda} = \frac{1}{2} f_2^n.$$

When Eqs. (6) are replaced into Eqs. (5), the theorem follows for WRDH, namely, $C_1 = C_2 = \cdots = C_5 = 0$.

A few remarks are in order. Notice that in the above discussions $\sigma \neq 0$ has been maintained. This leads to another way to put the theorem. One may restate it by saying: even if a priori mixings of positive-parity but flavor-violating eigenstates are allowed in physical hadrons (that is, $\sigma \neq 0$) the mixing angle σ will drop out of the matrix elements of the Yukawa and electromagnetic hamiltonian that lead to NLDH and WRDH in the strong flavor symmetry limit.

Concerning the parity-violating amplitudes A and D, no equivalent theorem seems to exist. In the flavor symmetry limit, the only possibility to make these amplitudes vanish is to required that the a priori mixing angles δ and δ' be put equal to zero from the outset. This is analogous to the case of assuming parity conservation; if indeed parity is a conserved quantum number one has no choice but to enforce $\delta = \delta' = 0$. There is, however, an exception in WRDH. It refers to the charge form factors f_1. These form factors are directly governed by the charge operator, which is a combination of generators that is always conserved. Once the f_1's are identified with the charges of the physical states, then the matrix elements of WRDH where they (the f_1's) appear vanish, even if $\delta,\ \delta' \neq 0$. This is the reason why the f_1's do not appear in the D amplitudes of Eqs. (6). One may then conclude that the parts of parity-violating amplitudes directly governed by conserved generators of the flavor-symmetry group will be zero even if non-zero parity-violating a priori mixings assumed to exist in hadrons [6].

One should contrast the above theorem with the existing theorems for the W_μ-mediated NLDH and WRDH, which are refered to as the Lee-Swift [7] and Hara [8] theorems, respectively. Both these theorems state that in the flavor symmetry limit it is the corresponding parity-violating amplitudes, A and D that vanish [9]. This illustrates how different are the a priori mixings approach and the ordinary W_μ-mediated approach to NLDH and WRDH.

There is another theorem available, but due to our current inability to compute well with QCD, we our unable to establish a direct connection with it. This is the so-called Feinberg-Kabir-Weinberg (FKW) theorem [10] or, more properly, its extension to quarks. This extension basically says that if one performes rotations to diagonalize the quark mass matrix to obtain off-shell quarks, then the rotation angles drop out, and accordingly an

absolutely conserved quantum number can be redefined, say, strangeness. The rotations considered in this theorem refer to equal parity (positive) quarks, mixings with opposite parity quarks have not yet been considered. The reason why the mixing angles drop out can be traced in the quark lagrangian to the flavor invariance of the QCD part and to the point-like e. m. coupling (only f_1-type couplings) of the quarks [11]. In this perspective, the present theorem might be seen to be the analogous of the FKW theorem, but this time at the hadron level. One can also see that at this level the equivalent of the FKW theorem will not eliminate the a priori mixing angles in general, because hadrons have more interactions than quarks and of those interactions the ones that are symmetry-breaking and not directly controlled by some conserved combination of generators make the mixing angles to give non-zero observable contributions. Eqs. (3) and (5) are examples of this. In addition, one can see that another reason for the FKW theorem not to eliminate the a priori mixing angles at the hadron level is that this theorem is valid for on-shell quarks and that the passage from the quark to the hadron level involves off-shell quarks, which are also in a highly non-perturbative regime, inside hadrons. It is probably for these reasons that it is extremely difficult to prove the FKW at the hadron level, starting from the quark level. However, it does not seem unreasonable for us to believe that the theorem we have discussed in this paper is somehow related to the FKW theorem.

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